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Bound solitons in a nonlinear optical coupler

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It is predicted that a nonlinear optical coupler in the form of a twin-core fiber with a frequency mismatch between the cores, or two parallel planar light guides carrying nonparaxial beams, should produce a set of stable bound states of solitons with different separations between their centers. Unlike the recently considered bound states of far separated optical solitons in single-mode and bimodal fibers, with an exponentially small binding energy, in the present case the bound states are robust (their binding energy has no exponential smallness), and the corresponding separation can be comparable to the proper size of the solitons. The predicted bound states can be used in the design of soliton-based memory and data-processing devices.

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Recently, bound states (BS's) of solitons in nonlinear systems governed by equations of the nonlinear Schrödinger (NLS) type have attracted considerable attention [1–6]. There are at least two important classes of physical systems in which the BS's of solitons (solitary pulses) can be predicted: nonlinear fiber and planar light guides [1–5], and traveling-wave convection in a narrow channel [7]. Actually, only in the latter system were the BS's observed in the experiment [7], while the solitonic BS's in the abovementioned nonlinear optical systems thus far remained a purely theoretical object. There is a fundamental difficulty impeding an easy observation of the BS's in the optical systems: as was shown in Ref. [1], in the case when the coefficients in front of the gain and dissipation terms in the corresponding perturbed NLS equation are small, which is the case in nonlinear optics, a BS of two solitons can be formed only when the separation between them is much larger than their proper size, the binding energy of this BS being exponentially small, so that an additional weak perturbation (e.g., the intrapulse stimulated Raman scattering [4]) can easily destroy them.

Of course, the BS may be more robust when the perturbation is stronger. In Ref. [6], results of very accurate numerical simulations, specially aimed at searching for the BS's, were reported for a known model (the ac-driven damped NLS equation, which has applications in solid state physics [8] and in nonlinear optics [5]). While for really small values of the perturbation coefficients it was virtually impossible to produce the BS, it was found at larger values of the coefficients (e.g., when a dimensionless dissipative constant was 0.6). Although perturbation theory, strictly speaking, does not apply in this case, it was shown that the BS's numerically found in this parametric region are still quite accurately described by the perturbative analytical expressions.

Coming back to nonlinear optical systems, it is relevant to mention that, as was shown in Ref. [3], in a system of two coupled NLS equations (which is a model of a bimodal optical fiber sensitive to the polarization of light), a BS of solitons belonging to different modes is possible without gain and dissipation, provided that the coherent nonlinear coupling between the modes and the group-velocity birefringence are taken into account. However, in this case, too, the separation between the bound solitons had to be large, and the binding energy was exponentially small. On the contrary to all this, the effective Ginzburg-Landau (perturbed NLS) equation which governs evolution of convection pulses in the narrow channel is known to have a large diffusion coefficient [9], which may be an explanation for a fairly robust BS observed in the channel [7]. The separation between the two "solitons" (subcritical convection pulses) in this BS was comparable to the pulse's proper size, which should be expected in this case.

The objective of the present work is to consider a BS of solitons in another bimodal nonlinear optical system, in which they are expected to be robust and much easier to observe experimentally. This is a twin-core fiber (nonlinear optical coupler [10]). Recently, a lot of work has been done to study soliton dynamics in couplers; see, e.g., Ref. [11]. In particular, interactions between the solitons belonging to the different cores were simulated numerically in Ref. [12]. One needs two essential ingredients to get a BS of solitons in a bimodal system [3]: a coherent (phase-sensitive) coupling between the modes, and a frequency mismatch between them. The linear coherent coupling is always present in the dual-core fiber. The frequency mismatch can be obtained if one simply launches in the two cores pulses with different carrier frequencies; to strengthen the *effective* mismatch, one may also use a coupler with asymmetric cores. In the most general case, the coupled NLS equations for the envelopes u and v of the electromagnetic waves in the two cores take the form

$$iu_z+\tfrac{1}{2}u_{\tau\tau}+|u|^2u = -Kv\, e^{-ikz+i\omega\tau}, \qquad (1)$$

$$\alpha(iv_z-icv_\tau+\tfrac{1}{2}\beta v_{\tau\tau}+\gamma|v|^2v) = -Ku\, e^{ikz-i\omega\tau}. \qquad (2)$$

Here z is the propagation distance, $\tau\equiv t-z/V_{\rm gr}$ is the reduced time chosen so that $V_{\rm gr}$ is the group velocity corresponding to the carrier wave in the first core, k, ω, and c are, respectively, the differences in the wave numbers, frequen-

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cies, and the group velocities between the carrier waves in the two cores, K is the coupling constant, β and γ are ratios between effective dispersion and Kerr coefficients in the two cores, and α is an additional possible asymmetry factor. The three coefficients α, β, and γ are assumed positive. Notice that the chosen form of Eqs. (1) and (2), with no asymmetry coefficient in front of the coupling terms, provides that the equations have a natural Hamiltonian; see below.

The model based on Eqs. (1) and (2) may also be applied to a pair of tunnel-coupled parallel nonlinear planar light guides, if the temporal variable τ is replaced by the spatial transverse coordinate x. In this case, the term which was interpreted above as the group-velocity mismatch can be naturally produced by a difference in propagation directions of the two guided light beams, and no additional asymmetry between the light guides is necessary.

In this work, interaction between solitons residing in the two cores of the coupler will be considered by means of the standard perturbation theory, treating the linear coupling as a perturbation. Actually, this problem was already considered perturbatively [13], with the essential difference that the two modes were assumed fully identical. It was demonstrated that a stable BS of two solitons with coinciding centers was possible. From the viewpoint of the analysis developed later in Refs. [1–6], this BS is, in a sense, trivial. Nontrivial BS's are those in which the centers of the bound solitons are separated.

To analyze the interaction of the solitons in the two cores, in the lowest approximation the solitons are taken in the usual form corresponding to the decoupled Eqs. (1) and (2):

$$u = \eta_1 \operatorname{sech}(\eta_1 \tau) \exp[i\phi_1(z)], \tag{3}$$

$$v = \gamma^{-1/2}\eta_2 \operatorname{sech}[\beta^{-1/2}\eta_2(\tau+T)] \exp[ic\beta^{-1}\tau + i\phi_2(z)], \tag{4}$$

where $\eta_{1,2}$ are amplitudes of the two solitons (actually, quantities of physical interest are the peak powers $\eta_{1,2}^2$), T is the separation between their centers, and the phases $\phi_{1,2}$ evolve according to the equations

$$\frac{d\phi_1}{dz}=\frac{1}{2}\eta_1^2, \quad \frac{d\phi_2}{dz}=\frac{1}{2}\beta^{-1}c^2+\frac{1}{2}\beta^{-1}\eta_2^2. \tag{5}$$

The system of Eqs. (1) and (2) is conservative, the part of its Hamiltonian which accounts for the coupling being

$$U = -K\int_{-\infty}^{+\infty}(u^* v\, e^{-ikz+i\omega\tau}+uv^*\, e^{ikz-i\omega\tau})d\tau. \tag{6}$$

Inserting the lowest-order approximation (3) and (4) into this expression, one can find an effective *potential* of the interaction between the two solitons. Obviously, the potential may give rise to BS's provided that it does not depend on the evolutional variable z. As follows from Eqs. (3)–(6), this condition implies the following relation between the amplitudes η_1 and η_2:

$$\beta\eta_1^2-\eta_2^2 = -2\beta k + c^2. \tag{7}$$

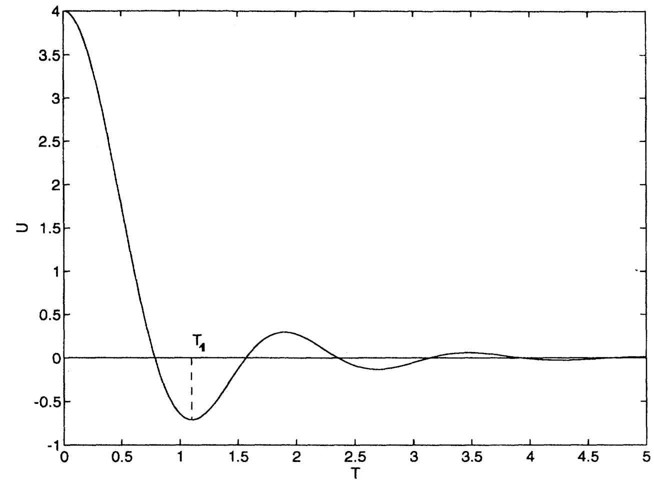

FIG. 1. The shape of the interaction potential (6) with $\phi=\pi$ and $\Omega=8\eta_1$.

If one takes initial values of the amplitudes not exactly satisfying the relation (7) but sufficiently close to it, and if the BS's produced by the potential (6) are stable (see below), it is natural to expect that the two solitons will indeed form one of the BS's, relaxing to it via emission of radiation, which is a generic scenario of reaching stable stationary states in conservative nonintegrable systems. However, a detailed analysis of the transient processes requires extensive numerical simulations and is left beyond the framework of this short communication.

Thus far, the analysis was carried out for the most general case amenable to application of perturbation theory. From now on, in order to obtain final results in a fully analytical form, attention will be confined to the particular case when both solitons (3) and (4) have the same internal scale, i.e., $\beta^{1/2}\eta_1=\eta_2$. It is necessary to stress that this limitation is singled out only by the purely technical condition that the integral (6) must be analytically calculable. There are no reasons to surmise that in the general case qualitative results will be essentially different from those to be presented below in an explicit form for this special case.

Setting in Eq. (6) $\beta^{1/2}\eta_1=\eta_2$, one can immediately perform the integration and obtain

$$U(T,\phi)=-4\pi\kappa\eta_1\cos\phi\,\frac{\sin(\Omega T/2)}{\sinh(\pi\Omega/2\eta_1)\,\sinh(\eta_1 T)}, \tag{8}$$

where $\kappa\equiv\gamma^{-1/2}K$, $\Omega\equiv\omega-\beta^{-1}c$ is an effective frequency mismatch between the cores, and $\phi\equiv\phi_1-\phi_2$ is the phase difference between the solitons, which remains constant in virtue of the relation (7). In the limit $\Omega=0$, the expression (8) goes over into the interaction potential obtained in Ref. [13] for the coupler with fully identical cores. For the sake of definiteness, the effective coupling constant κ will be assumed positive in what follows below.

The shape of the potential (8) is shown (for the π-out-of-phase solitons, i.e., $\phi=\pi$) in Fig. 1. To render the structure of the potential clearer, a large value of Ω, $\Omega=8\eta_1$, was taken in this figure. The BS's correspond to local minima of the potential [1] (the experience gained in analysis of similar problems [6] suggests that much shallower minima than those seen in Fig. 1, corresponding to essentially smaller values of Ω/η_1, should be able to support sufficiently stable BS's). Obviously, at $\phi=0$ the potential has the deepest minimum at $T=0$, which persists at $\Omega=0$ and corresponds to the well-known stable bound state of the two solitons with coinciding centers [13]. Nontrivial minima at finite separations T between the solitons exist only at $\Omega\neq 0$. It is straightforward to see that the location of the nontrivial minima is determined by the simple transcendental equation

$$\tan x = \xi \tanh(x/\xi) , \tag{9}$$

where $x\equiv\frac{1}{2}\Omega T$, and $\xi\equiv\Omega/2\eta_2$. Equation (9) gives rise to an infinite set of the minima, the first of them, T_1, lying in the interval

$$2\pi\Omega^{-1} < T_1 < 3\pi y_1\Omega^{-1}, \tag{10}$$

where $y_1\approx 0.96$ is the smallest positive root of the equation $\tan(\frac{3}{2}\pi y)=\frac{3}{2}\pi y$. It is easy to see that, in particular, T_1 is close to the left boundary of the interval (10) in the case $\Omega\ll\eta_1$, and it is close to the right boundary in the opposite case.

The minimum of the interaction potential at $T=T_1$ is attained when $\phi=\pi$, i.e., when the two solitons are π out of phase (Fig. 1). The second minimum is attained for the in-phase solitons, $\phi=0$, and then the values of ϕ equal to π and 0 alternate for subsequent minima. As follows from from Eq. (8), these positions are minima of the potential with respect to variation of both the separation T between the solitons and their phase difference ϕ. As the soliton's peak power (squared amplitude) may be regarded as a canonical momentum conjugate to the soliton's phase [see Eqs. (5)], a minimum with respect to variation of the phase difference ϕ implies that the corresponding BS will be stable against small perturbations of the solitons' amplitudes breaking the underlying relation (7) between them [1,3] (as well as stable against small perturbations of the separation T between the solitons).

At $\Omega\rightarrow 0$, all the nontrivial minima tend to infinity; see Eq. (10). However, in the general case, $\Omega/\eta_1\sim 1$, the separation between the two solitons in the first few BS's is of the same order of magnitude as the solitons' proper size $\sim\eta^{-1}$. As follows from Eq. (6), in this case there is no other smallness in the BS's binding energy but the small coupling constant K. This is a drastic difference from the BS's previously found in other single-component and two-component models of the nonlinear optical fibers [1,3], where the separation was much larger than the soliton's size, and the binding energy was exponentially small in the perturbation parameter. Thus the nontrivial BS's of solitons in the coupler with an effective frequency mismatch should be much more robust, and, therefore, they have a much better chance of being found in experiments with dual-core nonlinear fibers or dual-core nonlinear planar light guides.

In conclusion, it is pertinent to mention that the predicted BS's of the solitons may find an application in the design of nonlinear optical memory and logic chips. Obviously, the stable robust BS's with the separation between the solitons taking well-defined discrete values, which are not too large, can be used for storage of information (using usual BS's of the optical solitons in a single-mode pumped lossy fiber for information storage was recently proposed in Ref. [5]). Moreover, one can easily switch the states corresponding to the different separations, launching an additional free soliton in either core. A collision between the free soliton and the bound one will give rise to shifts of the location and phase of the latter soliton, which can switch it from one BS into another. The corresponding shifts are well predictable and easily controllable functions of parameters of the free soliton.

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